

Answers to Problem Set 4

Total: 35 marks

1.

$\neg \text{Cube}(a) \vee \text{FrontOf}(a, b)$
$\text{SameRow}(b, c) \wedge (\text{Dodec}(a) \vee \text{Cube}(a))$
$\text{Dodec}(a) \vee \text{BackOf}(c, a)$

Proof: [5 marks]

We will prove the conclusion by cases, based on the first premise.

Case (i) Suppose $\neg \text{Cube}(a)$. Now P2 entails that a is either a cube or a dodec. Hence $\text{Dodec}(a)$, and so a is either a dodec or c is back of it.

Case (ii) Suppose $\text{FrontOf}(a, b)$. P2 also tells us that b is in the same row as c , and so $\text{FrontOf}(a, c)$, which is equivalent to $\text{BackOf}(c, a)$. Hence either $\text{Dodec}(a)$ or $\text{BackOf}(c, a)$.

Since the conclusion holds in both cases, it holds in fact. ■

2. *Proof:* [5 marks]

We will prove the conclusion by reductio. Let us assume that the conclusion is false, i.e. a isn't a dodec, and c is not back of a . Now P2 tells us that a is either a dodec or a cube, so since a isn't a dodec we infer that a is a cube. But then P1 entails that a is front of b . Further, since b and c are in the same row, clearly a is front of c . But if a is front of c then c is back of a , and this contradicts the reductio assumption. Hence that assumption is false, and the conclusion follows. ■

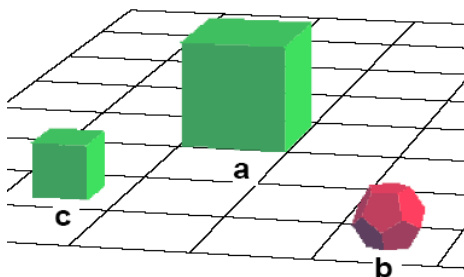
6.4 [5 marks]

1. $(A \wedge B) \vee C$	
2. $\nabla A \wedge B$	
3. B	✓ $\nabla \wedge$ Elim: 2
4. $C \vee B$	✓ $\nabla \vee$ Intro: 3
5. ∇C	
6. $C \vee B$	✓ $\nabla \vee$ Intro: 5
7. $C \vee B$	✓ $\nabla \vee$ Elim: 1,2-4,5-6

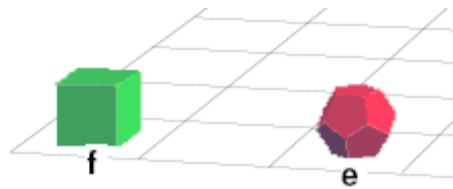
6.6 [5 marks]

1. $(A \wedge B) \vee (A \wedge C)$	
2. $\nabla A \wedge B$	
3. A	✓ $\nabla \wedge$ Elim: 2
4. B	✓ $\nabla \wedge$ Elim: 2
5. $B \vee C$	✓ $\nabla \vee$ Intro: 4
6. $A \wedge (B \vee C)$	✓ $\nabla \wedge$ Intro: 3,5
7. $\nabla A \wedge C$	
8. A	✓ $\nabla \wedge$ Elim: 7
9. C	✓ $\nabla \wedge$ Elim: 7
10. $B \vee C$	✓ $\nabla \vee$ Intro: 9
11. $A \wedge (B \vee C)$	✓ $\nabla \wedge$ Intro: 8,10
12. $A \wedge (B \vee C)$	✓ $\nabla \vee$ Elim: 1,2-6,7-11

6.10 The argument is invalid. [5 marks]



6.11 The argument is invalid. [5 marks]



6.24 [5 marks]

(N.B. The question asks for both formal and informal proofs for this argument, but just grade the formal proof.)

1. $\neg(A \vee B)$	
2. ∇A	
3. $A \vee B$	✓ $\nabla \vee$ Intro: 2
4. \perp	✓ $\nabla \perp$ Intro: 3,1
5. $\neg A$	✓ $\nabla \neg$ Intro: 2-4
6. ∇B	
7. $A \vee B$	✓ $\nabla \vee$ Intro: 6
8. \perp	✓ $\nabla \perp$ Intro: 7,1
9. $\neg B$	✓ $\nabla \neg$ Intro: 6-8
10. $\neg A \wedge \neg B$	✓ $\nabla \wedge$ Intro: 5,9